Mixed-mode fracture characterization by coupling digital images correlation with finite elements method

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Abstract:

This paper develops a new formalism based on a coupling between the experimental measurements by digital images correlation and a numerical approach by finite element method in order to evaluate the fracture parameters in wood material subjected to mixed-mode. The formalism proves that mixed-mode energy release rate can be evaluated from the Crack Relative Displacement and the Stress Intensity Factors without the knowledge of material mechanical properties. In the present work, the Crack Relative Displacement Factors are evaluated from the displacement field distribution in vicinity of the crack tip. Then, the Stress Intensity Factors are calculated from a finite element analysis. The experimental tests were realized using a Single Edge Notch specimen made in Douglas fir subject to mixed-mode loading.

Key words: Wood, Fracture Mechanics, Digital Images Correlation, Finite Elements, Energy release rate, Mixed-mode

1 Introduction

The fracture process under mixed-mode conditions is an important topic in timber structural durability. It is knows that the cracks occurrence, during the timber structure's life cycle, induces a stress concentration in the crack tip vicinity and affects the timber strength. Several approaches have been carried out in the literature for the purpose of characterizing crack tip parameters through use of the energy method for mixed-mode configurations for isotropic and orthotropic media [1], [2], [3], [4], [5], [6], [7]. At present, these techniques require the knowledge of material properties; for orthotropic cases in particular, the complete compliance tensor is needed. In this context and in order to circumvent these difficulties, the proposed method utilizes the combined digital image correlation measurements and finite element analysis. Since the digital image correlation has been widely used for full-field deformation measurements in the field of experimental mechanics, the application of such an experimental technique enables capturing both strong and weak kinematic discontinuities in the crack tip vicinity so as to characterize Crack Relative Displacement Factors (CRDF). According to the M-theta method, Stress Intensity Factors (SIF) are defined by introducing a mixed-mode separation algorithm. For orthotropic configurations, stress distribution into the crack tip domain is assumed to be unaffected by elastic properties. By aggregating the results of CRDF yielded by experimental DIC and SIF, as given by a numerical FEM, the prediction of an accurate energy release rate can be proposed. Fracture mode separation is also analyzed by establishing mode I and mode II energy release rates for mixed-mode loading conditions as well as stress plane configurations.

2 A new formalism in energy release rate estimation

The energy release rate fracture parameter can be estimate from different approaches using the local or global mechanical fields. However, the energy release rate estimation requests usually the knowing of elastic properties of material. In these conditions, we propose to develop a new formalism to estimate the energy release rate without the knowledge of material mechanical properties. So, in our approach, the energy release rate is estimated from four fracture parameters the Crack Relative Displacement Factors proposed by Dubois et al. [8], [9], [10] and the Stress Intensity Factors, as fallowing:

$$G_\alpha = \frac{K_\alpha^{(\varepsilon)} \cdot K_\alpha^{(\sigma)}}{8} \quad (1)$$

By using the expression (1), the energy release rate can be estimated by coupling the experimental measurement with the numerical analysis by finite element method. Then, the Crack Relative Displacement Factors $K_\alpha^{(\varepsilon)}$ (where α is the fracture mode I or II) are estimated from the kinematic state in the crack tip vicinity. In our approach, the kinematic state was defined from the experimental displacement fields measured by Digital Image Correlation.

The second fracture parameters, the Stress Intensity Factors $K_\alpha^{(\sigma)}$, introduced in expression (1), is evaluated from a finite element analysis. In terms of mesh geometry and loading configuration, the approach is based on the experimental test configuration.

3 Experimental evaluation of the Crack Relative Displacement Factor

The experimental tests were carried out using the Single Edge Notch samples made in Douglas fir. The sample size is 210x150x10mm^3 with an initial crack length equal to 90 mm. Note also that the crack is parallel with the grain orientation. The sample moisture content is fixed at 11% into a conditionnement climatic room, before the test.

The loading configuration is defined by the angle measured between the loading and the crack directions. In the present work, three mixed-mode configurations, corresponding to 15°, 45° and 75°, are tested. The tests were realized using an electromechanically press and the various mixed mode configurations were applied using the Arcan fixtures (Figure 1).

During the tests, the mechanical behavior of samples was recorded using two measurement devices. So, a LVDT transducer and a load cell were used to estimate the global behavior by measuring the displacement of the loading point. The second device measures the displacement field's distribution in the crack tip vicinity. For this, a CCD camera is added in order to measure the sample behavior by Digital Image Correlation. After the test, the Crack Relative Displacement Factors are evaluated from experimental displacements. Then, for a given loading state, the displacement field is evaluated by comparing the initial image corresponding to undeformed state of sample with the corresponding deformed image, according with DIC method [11]. In order to estimate the displacement fields by DIC, the sample surface is covered with a black and white speckle pattern.

As shown in figure 1, according with DIC principle, the displacement field is calculated around to the crack tip into zone of interest discretized by the subsets. The displacement field corresponds to displacement vectors associated to the subset centers.

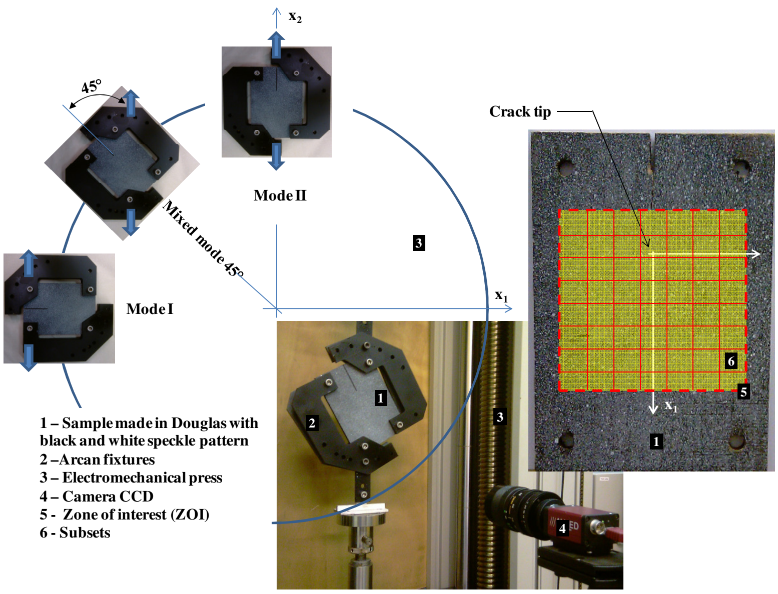

FIG. 1 – Wood specimen without random pattern and experimental setup

Now, from the experimental measurements by digital image correlation, the Crack Relative Displacement Factors (CRDF) evaluation can be realized by two ways. If the experimental data isn't affected by the experimental noises [12], then the CRDF estimation can be performed directly from the experimental data. Before to calculate the crack relative displacement factors, it is necessary to optimize the experimental measurements. We'll use this second way and the experimental data were optimized from an adjustment procedure. In our case, it is based on the Newton-Raphson algorithm and the non linear least squares method. The optimized fields were obtained by adjusting the asymptotic William's series (2) [6], [8] on the displacement fields obtained by DIC [6], [8], [13].

$$
\begin{aligned}
u_1^k &= \sum_{i=1}^{N}\left(A_I^i \cdot r_k^{(i/2)} \cdot f_i\left(\kappa,\theta_k\right) + A_{II}^i \cdot r_k^{(i/2)} \cdot g_i\left(\kappa,\theta_k\right)\right) + T_1 - R \cdot x_2^k \\
u_2^k &= \sum_{i=1}^{N}\left(A_I^i \cdot r_k^{(i/2)} \cdot l_i\left(\kappa,\theta_k\right) + A_{II}^i \cdot r_k^{(i/2)} \cdot z_i\left(\kappa,\theta_k\right)\right) + T_2 + R \cdot x_1^k
\end{aligned} \tag{2}
$$

In order to take into account the experimental boundaries conditions, the rigid body motions components T_1, T_2 and R are added in analytical expressions (2).

N is the power series number. κ and A_α^i are the weighting coefficients with α = I, II and i = 1..N. $f_i(\kappa,\theta)$, $g_i(\kappa,\theta)$, $l_i(\kappa,\theta)$ and $z_i(\kappa,\theta)$ are the polar functions defined by:

$$
\begin{aligned}
f_i(\kappa,\theta) &= \kappa \cdot cos\left(\frac{i}{2}\cdot\theta\right) - \frac{i}{2}\cdot cos\left(\frac{i}{2}-2\right)\cdot\theta + \left\{\frac{i}{2}+(-1)^i\right\}\cdot cos\left(\frac{i}{2}\cdot\theta\right) \\
g_i(\kappa,\theta) &= -\kappa \cdot sin\left(\frac{i}{2}\cdot\theta\right) + \frac{i}{2}\cdot sin\left(\frac{i}{2}-2\right)\cdot\theta - \left\{\frac{i}{2}-(-1)^i\right\}\cdot sin\left(\frac{i}{2}\cdot\theta\right) \\
l_i(\kappa,\theta) &= \kappa \cdot sin\left(\frac{i}{2}\cdot\theta\right) + \frac{i}{2}\cdot sin\left(\frac{i}{2}-2\right)\cdot\theta - \left\{\frac{i}{2}+(-1)^i\right\}\cdot sin\left(\frac{i}{2}\cdot\theta\right) \\
z_i(\kappa,\theta) &= +\kappa \cdot cos\left(\frac{i}{2}\cdot\theta\right) + \frac{i}{2}\cdot cos\left(\frac{i}{2}-2\right)\cdot\theta - \left\{\frac{i}{2}-(-1)^i\right\}\cdot cos\left(\frac{i}{2}\cdot\theta\right)
\end{aligned} \tag{3}
$$

In order to take into account the experimental boundaries conditions in analytical expression (3). Another aspect, taken into account in the optimization procedure, is the position and orientation of the crack tip.

$$
r_k = \sqrt{\left(x_1^k - x_1^o\right)^2 + \left(x_2^k - x_2^o\right)^2} \; and \; \theta_k = \tan^{-1}\left(\frac{x_2^k - x_2^o}{x_1^k - x_1^o}\right) - \omega_o \tag{4}
$$

x_1^0, x_1^0 and ω_0 are the initial position and orientation of the crack tip. x_1^o and x_2^o are the crack tip location relative to an arbitrary coordinate system and ω_o being an overall crack, respectively.

Once the experimental displacement has been optimized, the CRDF can be estimated. Based on the Williams serie (2), Dubois has established a relationship between the Crack Relative Displacement Factors and the first weighting coefficients corresponding to each mode and the parameter κ [8], [9], [10], as following:

$$
K_I^{(\varepsilon)} = 2 \cdot A_I^1 \cdot \sqrt{2\cdot\pi} \cdot (\kappa+1) \; and \; K_{II}^{(\varepsilon)} = -2 \cdot A_{II}^1 \cdot \sqrt{2\cdot\pi} \cdot (\kappa+1) \tag{5}
$$

We remind that the weighting coefficients and the parameter κ introduced in expression (5) are obtained from the adjustment procedure. As can be observed, the CRDF evaluation is realized from the experimental measurement without using elastic properties of the material.

4 Numerical evaluation of the Stress Intensity Factor

In our approach, the Stress Intensity Factors (SIF) evaluation is performed from a finite element analysis. As indicated above, our purpose is to evaluate the energy release rate without elastic properties of the material. So, the Stress Intensity Factor evaluation should be carried out without using the elastic properties of the material. For a loading configuration under force control, Pop et al. [6] and Dubois et al. [8] have shown that the SIF's are invariant to elastic properties of material and there evaluation can be performed using arbitrary elastic properties. However, in the orthotropic case, this aspect is verified only if the orthotropic ratio remains constant.

The finite element analysis is based on the experimental test configurations. So the mesh geometry, the loading and the boundary conditions correspond to experimental configuration. The numerical model is schematized in figure 2.

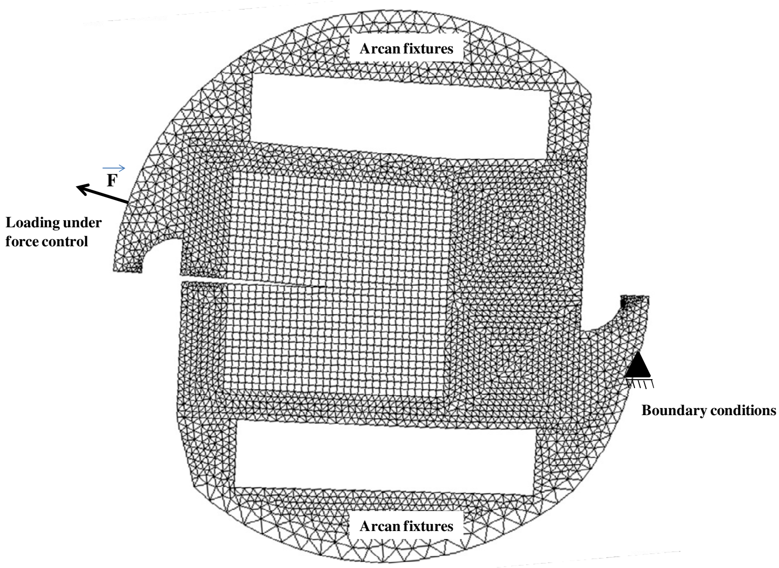

FIG. 2 – Finite element model

From the finite element analysis, the Stress Intensity Factors, corresponding to opening and shear modes are estimated by using the integral invariant M-theta [5], [6], [8] such as:

$$ {}^{u}K_{I}^{(\sigma)} = \frac{8\cdot \tilde{M}\theta_{I} \cdot \left({}^{v}K_{I}^{(\sigma)} = 1, {}^{v}K_{II}^{(\sigma)} = 0 \right)}{\tilde{C}_{I}} \text{ and } {}^{u}K_{II}^{(\sigma)} = \frac{8\cdot \tilde{M}\theta_{II} \cdot \left({}^{v}K_{I}^{(\sigma)} = 1, {}^{v}K_{II}^{(\sigma)} = 0 \right)}{\tilde{C}_{II}} \qquad (6) $$

" ~ " indicated that parameters are calculated with arbitrary elastic properties. $\tilde{C}_{\alpha}$ is the arbitrary reduced elastic compliance [6], [8]. The superscript indices "u" corresponds to real mechanical field, while "v" corresponds to auxiliary mechanical fields [8].

5 Results and discussions

During the test, the sample behavior remains elastic and the crack is stationary. Note also that, in the present work, only one loading state is analyzed, for each mixed-mode configurations (see table 1). After test, the experimental noised data have been optimized by adjusting the asymptotic Williams series. The figures 3 and 4 show the adjustment procedure results, by illustrating a comparison between experimental and optimized displacement fields.

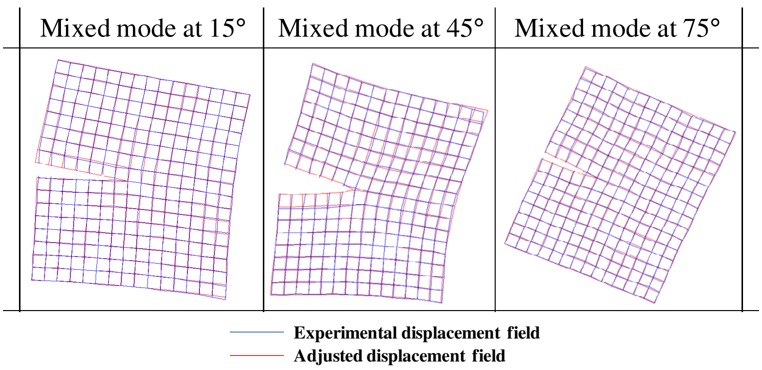

FIG. 3 – Comparison between the experimental and the optimized fields

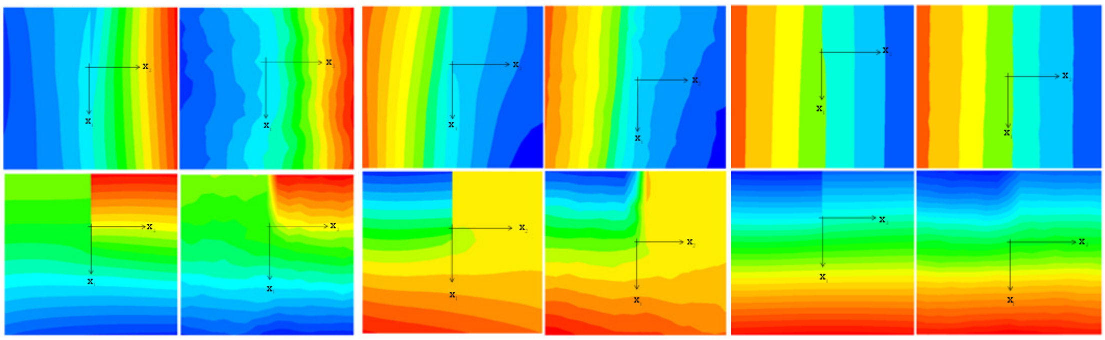

FIG. 4 – Comparison between the experimental and the adjusted fields (up displacement in direction x_1, down up displacement in direction x_2)

The two comparisons show a good agreement.

Now, using the optimized displacements fields the CRDF, corresponding to each fracture mode, are evaluated by replacing the numerical values of the weighting coefficients A^1_α and the κ parameter in equation (5). The CRDF values calculated from equation (5) are resumed in table 1.

Mixed mode	Applied load (N)	$K_I^{(\varepsilon)}$ ($m^{1/2}$)	$K_{II}^{(\varepsilon)}$ ($m^{1/2}$)
15°	277	$5.95 \cdot 10^{-4}$	$1.43 \cdot 10^{-4}$
45°	784	$1.35 \cdot 10^{-3}$	$7.43 \cdot 10^{-4}$
75°	876	$6.91 \cdot 10^{-4}$	$6.58 \cdot 10^{-4}$

TABLE 1 – Crack relative displacement factor values

In parallel, from a finite element analysis, the SIF is evaluated using the iM-theta method. The orthotropic ratios, associated to the arbitrary elastic properties, are those proposed by Guitar for a resinous [14]. The Stress Intensity Factor values are resumed in table 2.

Mixed mode	Applied load (N)	$K_I^{(\sigma)}$ ($MPa \cdot m^{1/2}$)	$K_{II}^{(\sigma)}$ ($MPa \cdot m^{1/2}$)
15°	277	0.14	0.11
45°	784	0.32	0.59
75°	876	0.16	0.52

TABLE 2 – Stress intensity factor values

By replacing the CRDF and SIF values in expression (1), the energy release rate can be calculated without the real elastic properties of the material. These values are resumed in table 3.

Mixed mode	Applied load (N)	G_I (J/m^2)	G_{II} (J/m^2)
15°	277	10.49	2.02
45°	784	54	55.22
75°	876	14.16	42.88

TABLE 3 – Energy release rate values

6 Conclusion

This paper focus on a new method for the characterization of the mixed-mode energy release rate for orthotropic media through a coupling between the experimental and numerical approaches. Mixed-mode energy release rate uncoupling prediction is achieved by combining two crack tip parameters, the Crack Relative Displacement Factors and Stress Intensity Factors. The first is defined from the kinematic state and the displacement field measured by Digital Images Correlation in the crack tip vicinity. While the second fracture parameters is calculated from numerical modeling by the Finite Element Method and the invariant integral M-theta. Thus, according with the new formalism, the energy release rate prediction, in mixed-mode fracture, is possible without the knowledge of material mechanical properties.

Crack initiation and propagation can further study from this new technique in order to apprehend all phenomenon governing the crack tip growth process. A generalization of such technique to others law behavior such as visco-elasticity, visco-plasticity, and plasticity can be further expected.

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